Force / Motion Control for Constrained Robot Manipulator Using Adaptive Neural Fuzzy Inference System (ANFIS)

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ABSTRACT

This paper presents an Adaptive Neural Fuzzy Inference System (ANFIS) control for constrained robot manipulator to compensate the uncertainties in the robot dynamics. ANFIS is trained online based on computed torque method on the dynamic system to survey a desired both force and motion trajectory. The mathematical equations of the control law and closed loop errors are derived. Based on the derived equations the ANFIS is used to identify the dynamic parameters of constrained robot. The simulation is carried out using a two link constrained robot. Simulation results show that the trajectory tracking error of motion and force are converged with good accuracy.

1. Introduction

Industrial robots are widely used in different branches of industry. Some of their applications are spray painting, arc and spot welding, injection, handling all these applications are simple and repetitive, and what they all have in common is that these robots follow only position commands for the execution of these tasks.[1] On the other hand, applications require the control of not only the position of the manipulator, but also the force applied by the robot on the objects, such that the end-effector exerts desired force to the environment as the robot moves along a prescribed trajectory. Typical tasks of this kind include grinding, polishing, inserting, scribing, digging, cutting, and assembly tasks... etc. The robot can be either unconstrained if there is no physical interaction between the end-effector and the environment, or constrained if contact forces arise between the end-effector and the environment.[2] Constrained robots have become a useful mathematical method to model the physical and dynamic effects of a robot when it engaged in one of the contact tasks. Unlike free motion control, where the only control objective is trajectory tracking or set point regulation. In automated operation of such process, the motion and the tangential velocity of the tool along the work piece and the force normal to the work surface need to be controlled. This is difficult achieve by position control alone, since uncertainties in the position of the work piece, and those in the stiffness of the tool holding device and manipulator. This research topic has received much attention in the robot control in last decade.

The ANFIS is a very powerful approach for modelling nonlinear and complex systems with less input and output training data and quicker learning and high precision. It has the advantages of expert knowledge of fuzzy logic and learning capability of neural networks [4]. Since there introducing by Jang [3], the ANFIS has been widely used to solve many problems in modelling and control. The objective of ANFIS is to optimize the parameters of a given fuzzy inference system by applying a learning procedure using a set of input-output training data. [5]
In this paper, firstly, the dynamic mathematical equations for constrained robot is presented to decouple the robot dynamics into two subsystems, motion subsystem and force subsystem. Then the hybrid force/motion control law is derived from dynamic equations. The design of ANFIS model structure is given with the studied two links constrained robot model.

2. Dynamic Equations of Constrained Robot

The dynamic equation of motion for constrained robot manipulator with n degree of freedom, taking into consideration the contact force and the constraints is given by:

\[ M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = U + J^T(q) \lambda \]  

(1)

Where \( M(q) \in \mathbb{R}^{n \times n} \) is the generalize inertia matrix of the system, \( C(q, \dot{q}) \in \mathbb{R}^{n \times n} \) is the matrix of centrifugal and coriolis terms. \( G(q) \in \mathbb{R}^n \) is the vector of the gravitation term, \( U \in \mathbb{R}^n \) denotes the vector of generalize input torque \( \lambda \in \mathbb{R}^m \) is the contact force such that the constrained forces are associated with this vector by:

\[ f = J^T(q) \lambda \]  

(2)

Where \( J \) is the Jacobean matrix

The vector \( (q) \in \mathbb{R}^n \) denotes the generalized displacement of \( n \)-degree of freedom robotic manipulator

The motion of constrained robot manipulator can be defined by an algebraic equation as: [6]

\[ \phi(q) = 0 \]  

(3)

Here \( \phi(q) \in \mathbb{R}^m \) is the vector of constrained function, represents the environment surface on which the manipulator end-effector lies, \( m \) is the number of constraints with \( m < n \).

By defining the vector partition \( q \) in two vectors, an \((m \times 1)\) vector \( q_1 \) and an \(((n - m) \times 1)\) vector \( q_2 \). This partition should be made so that the Jacobean matrix \( J(q) \) can be divided into two terms:

\[ J(q) = \begin{bmatrix} J_1(q) & J_2(q) \end{bmatrix} \]  

(4)

\[ J_1(q) = \frac{\partial \phi}{\partial q_1} ; \quad J_1(q) \in \mathbb{R}^{m \times m} \]  

(5)

\[ J_2(q) = \frac{\partial \phi}{\partial q_2} ; \quad J_2(q) \in \mathbb{R}^{m \times (n-m)} \]  

(6)

With rank of \( J_1 \) is \( m \)

Then the constraint equation (3) can be expressed in terms of variable \( q_2 \) only:

\[ q_1 = \psi(q_2) \Rightarrow \phi(\psi(q_2), q_2) = 0 \]  

(7)

Where \( \psi \) is a differentiable function.
Thus a nonlinear transformation can be introduced so that the equation of motion of the constrained robot can be written in a simpler form for ease of developing the control algorithms:

Define a vector $X$ and its partition as

$$x_1 \in R^m, \ x_2 \in R^{n-m}$$

For this: the change of coordinates is applied:

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} q_1 - \psi(q_2) \\ q_2 \end{bmatrix} = X(q) \tag{8}$$

Which is differentiable and has inverse transformation $Q(x)$:

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = Q(x) = \begin{bmatrix} x_1 + \psi(x_2) \\ x_2 \end{bmatrix} \tag{9}$$

Define also the differentiation of the above inverse transformation:

$$T(x) = \frac{\partial Q(x)}{\partial x} = \begin{bmatrix} I_m & \frac{\partial \psi(x_2)}{\partial x_2} \\ 0 & I_{n-m} \end{bmatrix} \tag{10}$$

In the new coordinates, satisfaction of the constraint equation (3) implies that:

$$x_1 = 0 \quad \text{and} \quad \dot{x}_1 = \ddot{x}_1 = 0 \tag{11}$$

Define the identity matrix partition

$$I_n = \begin{bmatrix} E_1^T & E_2^T \end{bmatrix} \quad \text{with:}$$

$$E_1 = \begin{bmatrix} I_m & 0 \end{bmatrix} \quad \text{and} \quad E_2 = \begin{bmatrix} 0 & I_{n-m} \end{bmatrix} \tag{12}$$

Then

$$X = \begin{bmatrix} 0 \\ x_2 \end{bmatrix} = E_2^T x_2 \tag{13}$$

$$\dot{X} = E_2^T \dot{x}_2 \tag{14}$$

$$\ddot{X} = E_2^T \ddot{x}_2 \tag{15}$$

From the inverse transformation equation (9) we will make use of the differential relation

$$\dot{q} = \begin{bmatrix} I_m & \frac{\partial \psi}{\partial x_2} \\ 0 & I_{n-m} \end{bmatrix} \dot{X} \tag{16}$$

$$\dot{q} = T(x_2) \dot{X} \tag{17}$$

and

$$\ddot{q} = T(x_2) \ddot{X} + \dddot{T}(x_2) \dot{X} \tag{18}$$
If we express the differential equation (1) in terms of the variables $X$ and pre-multiply both sides of that equation by $T^T(x_2)$, we obtain the following equation:

$$M^*(x_2)E_2^T \dot{x}_2 + C^*(x_2, \dot{x}_2)E_2^T \dot{x}_2 + G^*(x_2) = U^* + J^T(x_2)\lambda$$

(19)

Where

$$M^* = T^T(x_2)M(Q(x_2))T(x_2)$$

(20)

$$C^* = T^T(x_2)\left[ M(Q(x_2))\tilde{T}(x_1) + C(Q(x_2), T(x_1)\dot{x})T(x_2) \right]$$

(21)

$$G^* = T^T(x_2)G(Q(x_2))$$

(22)

$$J^* = T^T(x_2)J^T(Q(x_2))$$

(23)

$$U^* = T^T(x_2)U$$

(24)

Then equation (19) can be expressed in two subsystems of reduced from:

$$E_2M^*(x_2)E_2^T \dot{x}_2 + E_2C^*(x_2, \dot{x}_2)E_2^T \dot{x}_2 + E_2G^*(x_2) = E_2U^* + E_2J^T(x_2)\lambda$$

(25)

$$E_2M^*(x_2)E_2^T \dot{x}_2 + E_2C^*(x_2, \dot{x}_2)E_2^T \dot{x}_2 + E_2G^*(x_2) = E_2U^*$$

(26)

In obtaining equation (25) and equation (26) we have used the fact that:

$$E_2T^T J^T \lambda = 0$$

(27)

The differential equation (25) is $m$ system equations characterize the motion of the constrained robot system and equation (26) is $(n-m)$ system equations describes the contact force during the constrained motion, expressed in terms of motion and control input.

3. Hybrid Force/Motion Control Law

The control problem requires the tracking of the following motion and force trajectories $x = x_d$ and $\lambda = \lambda_d$.

Where $x$ and $x_d$ are the actual and desired motion trajectory. $\lambda$ and $\lambda_d$ are the actual and desired contact force respectively. The subsystem equation (25) and equation (26) will be used for deriving an inversion control law in computed torque method:

From the ($m$) equation (25), the control input ($U$) will be of the form:

$$E_1^T U = E_1 \hat{C}^* E_2^T \dot{x}_2 + E_1 \hat{G}^* + E_1 \hat{M}^* E_2^T u_{ox} - E_1 T^T J^T u_{o\lambda}$$

(29)

and the control input $U$ from $(n-m)$ equation (3.26) will be:

$$E_2^T U = E_2 \hat{C}^* E_2^T \dot{x}_2 + \hat{G}^* + E_2 \hat{M}^* E_2^T u_{ox}$$

(30)

Where $u_{ox}$ and $u_{o\lambda}$ are an auxiliary control inputs to be designed to ensure tracking for constrained motion and contact force and $\hat{M}$, $\hat{C}$ and $\hat{G}$ are estimated values of $M$, $C$ and $G$ because actually the dynamical parameters of robot manipulator are not known exactly.

Typical choices for these control inputs [7]:
\[ u_{na} = \ddot{x}_d + K_D (\dot{x}_d - \dot{x}) + k_p (x_d - x) \]  \hspace{1cm} (31)

\[ u_{na} = \dddot{\lambda}_d + K_D (\dot{\lambda}_d - \dot{\lambda}) + K_I \int_0^t (\lambda_d - \lambda) d\tau \]  \hspace{1cm} (32)

I.e. a proportional derivative (PD) action on the motion error has been added to the feedforward acceleration \( \ddot{x}_d \). While a proportion integral (PI) action was chosen on the force error, apart from the feedforward of \( \dot{\lambda}_d \). \( K_D, K_p, K_D, \) and \( K_I \) are positive constants. Combining \( n \) equations (29), (30) and solving for \( U \) gives the control law:

\[ U = T^T M^T E_2^T (\ddot{x}_{2d} + K_D (\dot{x}_{2d} - \dot{x}_2) + K_p (x_{2d} - x_2)) + T^T (\dddot{\lambda}^* E_2^T \dot{x}_2 + \dddot{G}^*) \]

\[ - J^T \left( \dddot{\lambda}_d + K_D (\dot{\lambda}_d - \dot{\lambda}) + K_I \int_0^t (\lambda_d - \lambda) d\tau \right) \]  \hspace{1cm} (33)

The control law of above equation can be written in a terms of \( M, C \) and \( G \) instead of \( M^*, C^*, G^* \):

\[ U = \dot{M} T E_2^T (\ddot{x}_{2d} + K_D (\dot{x}_{2d} - \dot{x}_2) + K_p (x_{2d} - x_2)) + \dot{C} T E_2^T \dot{x}_2 \]

\[ + \dot{G} + \dot{M} T E_2^T \dot{x}_2 - J^T \left( \dddot{\lambda}_d + K_D (\dot{\lambda}_d - \dot{\lambda}) + K_I \int_0^t (\lambda_d - \lambda) d\tau \right) \]  \hspace{1cm} (34)

Equation (33) or equation (34) represents the hybrid force/motion control law for constrained robot.

### 3.1. Error System Analysis

The analysis of the characteristics of the closed-loop error system is made in terms of the following errors:

\[ e_0 = x_1 = q_1 - \psi(q_2) = 0 \]  \hspace{1cm} (35)

\[ e_1 = x_2 - x_{2d} = q_2 - q_{2d} \]  \hspace{1cm} (36)

\[ e_2 = \dot{\lambda} - \dot{\lambda}_d \]  \hspace{1cm} (37)

Substituting the control law equation(34) into equations (25) and (26) we obtain:

\[ \dddot{e}_1 + K_D \dddot{e}_1 + K_p e_1 = (E_2 T^T \dot{M} T E_2^T)^{-1} \]

\[ \left[ E_2 T^T (\Delta M T E_2^T \dot{x}_2 + \Delta M \dot{T} E_2^T \dot{x}_2 + \Delta C T E_2^T \dot{x}_2 + \Delta G) \right] \]  \hspace{1cm} (38)

\[ E_2 T^T J^T (e_2 + K_D e_2 + K_I \int_0^t e_2 d\tau) = (\Delta M T E_2^T \dot{x}_2 + \Delta M \dot{T} E_2^T \dot{x}_2 + \Delta C T E_2^T \dot{x}_2 + \Delta G) \]

\[ \left[ E_2 T^T - E_2 T^T \dot{M} T E_2^T (E_2 T^T \dot{M} T E_2^T)^{-1} E_2 T^T \right] \]  \hspace{1cm} (39)

Where

\[ \Delta M = \dot{M} - \dot{M}, \quad \Delta C = \dot{C} \quad \text{and} \quad \Delta G = \dot{G} \]  \hspace{1cm} (40)

It is clear from equation (38) and equation (39) when:

\[ M = \dot{M}, \quad C = \dot{C} \quad \text{and} \quad G = \dot{G} \]  \hspace{1cm} (41)

the equations will be:

\[ \dddot{e}_1 + K_D \dddot{e}_1 + K_p e_1 = 0 \]  \hspace{1cm} (41)
\[ e_2 + K_d e_2 + K_p \int_0^t e_2 \, d\tau = 0 \quad (42) \]

However, generally \( M \neq \dot{M}, \ C \neq \dot{C} \) and \( G \neq \dot{G} \) and according to equations (38), (39) selecting \((K_p, K_d, K_\dot{q}, \text{ and } K_t) \in R^2\) is not sufficiently guarantee to achieve a proper response which means a response with the minimum acceptable dynamic error. Thus to achieve a proper response the dynamical parameters of the constrained robot should be identified precisely.

### 4 Design of The ANFIS

Based on the equations (38) and (39), ANFIS network is designed to identify the dynamical parameters of the constrained robot on-line such that the dynamical position error \((\dot{e}_i + K_d \dot{e}_i + K_p e_i)\) and the dynamical force error \((e_2 + K_\dot{q} e_2 + K_t \int e_2 \, d\tau)\) are approached to zero.

#### 4.1 Studied Robot Model

Consider a two link robot manipulator with a circular path constrained. The original model of the system is re-arranged in terms of the following components:

\[
M(q) = \begin{bmatrix}
b & b + c \cos(q_1) \\
b + c \cos(q_1) & a + b + 2c \cos(q_1)
\end{bmatrix} \quad (43)
\]

\[
C(q, \dot{q}) = \begin{bmatrix}
0 & c \dot{q}_2 \sin(q_1) \\
-c(\dot{q}_1 + \dot{q}_2) \sin(q_1) & -c \dot{q}_1 \sin(q_1)
\end{bmatrix} \quad (44)
\]

\[
G(q) = 9.81 \begin{bmatrix}
c \cos(q_1 + q_2) \\
a \cos(q_1) + c \cos(q_1 + q_2)
\end{bmatrix} \quad (45)
\]

Where \( a, b \) and \( c \) are uncertain physical parameters of the mechanical system:

\[
a = (m_1 + m_2) \ell_1^2, \quad b = m_2 \ell_2^2 \quad \text{and} \quad c = m_2 \ell_1 \ell_2
\]

Where \( \ell_1 \) and \( \ell_2 \) are the length of the two robot links, \( m_1 \) and \( m_2 \) are the mass of the two links. The constraint in \( P_1-P_2 \) Cartesian plane is supposed to be a circle whose centre coincides with the axis of rotation of the first link.

\[
\phi(p) = P_1^2 + P_2^2 - r^2 \quad (46)
\]

In this case \( \Delta M, \ \Delta C \) and \( \Delta G \) can be given as :

\[
\Delta M = \begin{bmatrix}
b - \hat{b} & (b - \hat{b}) + (c - \hat{c}) \cos(q_1) \\
(b - \hat{b}) + (c - \hat{c}) \cos(q_1) & (a - \hat{a}) + (b - \hat{b}) + 2(c - \hat{c}) \cos(q_1)
\end{bmatrix} \quad (47)
\]
\[
\Delta C = \begin{bmatrix}
0 \\
-(c-c)\dot{q}_2 \sin(q_1) \\
-(c-c)(\dot{q}_1 + \dot{q}_2)\sin(q_1)
\end{bmatrix}
\] (48)

\[
\Delta G = 9.81 \begin{bmatrix}
(c-c)\cos(q_1 + q_2) \\
(a-a)\cos(q_1) + (c-c)\cos(q_1 + q_2)
\end{bmatrix}
\] (49)

In these formulas \( \hat{a}, \hat{b} and \hat{c} \) will be an estimated amounts of \( a, b \) and \( c \) respectively, as its clear if \( a = \hat{a}, b = \hat{b} and c = \hat{c} \) the dynamical errors in equation(38) and equation(39) would be zero. The aim is to identify the dynamic parameters of the robot.

In this work, Three ANFIS models are used to identify the amounts of dynamic parameters \( a, b \) and \( c \) of the constraint robot according to the dynamical errors in equations (38) and (39). The ANFIS models should be trained according to the robot information and then used to estimate the acceptable amount of \( a, b \) and \( c \). To produce the training data the dynamical errors of the constrained robot are calculated by changing the amount of parameters \( a, b \) and \( c \).

A basic block diagram of the system and the controller with ANFIS is shown in Fig.(1).

![Fig. (1) Basic block diagram of the system and controller](image)

The design of ANFIS model structure is based on a first order Sugeno fuzzy model so the consequent part of the fuzzy If-Then rules is a linear equation. A generalized bell-shaped membership functions are selected for inputs. Seven membership functions are chosen on each input.

### 3.4 Simulation Results

The considered two link robot \( (n=2) \) with circular path of one constraint \( (m=1) \)
The constraint equation in \( P_1 - P_2 \) plane

\[ \phi(p) = P_1^2 + P_2^2 - r^2 \]

The transformation of the constraint equation to the joint variables of robot is

\[ P_1 = \ell_1 \cos q_1 + \ell_2 \cos (q_1 + q_2) \] (50)

\[ P_2 = \ell_1 \sin q_2 + \ell_2 \sin (q_1 + q_2) \] (51)

The transformation of the constraint equation from workspace to the joint space yields to the following constraint equation:

\[ \phi(q) = \ell_1^2 + \ell_2^2 + 2\ell_1\ell_2 \cos(q_1) - r^2 = 0 \] (52)

Which has a unique solution \( q_{i0} \) such that:

\[ q_{i0} = \psi(q_2) = \psi(x_2) = \cos^{-1} \left( \frac{r^2 - (\ell_1^2 + \ell_2^2)}{2\ell_1\ell_2} \right) \] (53)

The Jacobean of \( \phi(q) \) is then given as defined in equations (5), (6) as:

\[ J^T(q) = \begin{bmatrix} -2\ell_1\ell_2 \sin(q_{i0}) \\ 0 \end{bmatrix} \] (54)

The matrix \( T(x_2) \) defined in equation (10) is:

\[ T(x_2) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \] (55)

By applying the transformation of coordinates we obtain the following component \( M^* \) as defined in equation (20), then given as:

\[ M^* = \begin{bmatrix} b & b + c \cos(q_{i0}) \\ b + c \cos(q_{i0}) & a + b + 2c \cos(q_{i0}) \end{bmatrix} \] (56)

\( C^* \) is defined in equation (21), which given as:

\[ C^* = \begin{bmatrix} 0 & c \dot{x}_2 \sin(q_{i0}) \\ -c \dot{x}_2 \sin(q_{i0}) & 0 \end{bmatrix} \] (57)

\( G^* \) is defined in equation (22), given as:

\[ G^* = 9.81 \begin{bmatrix} c \cos(q_{i0} + x_2) \\ a \cos(q_{i0}) + c \cos(q_{i0} + x_2) \end{bmatrix} \] (58)
The parameters values are set to be:

\[
\ell_1 = 1m, \quad \ell_2 = 0.8m, \quad m_1 = 3.5Kg, \quad m_2 = 1.5Kg, \quad r=0.6,
\]

the controller gains are selected to be \( K_p = 10 \), \( K_D = 10 \), \( K_i = 2 \) and \( K_d = 0.2 \).

The desired trajectory is considered as the following

\[
q_{2d} = -\frac{\pi}{2} + 0.9 \left[ 1 - \cos(1.26t) \right]
\]

and the desired contact force is 20 N.

The nominal values of the dynamic parameters is taken as \( a_o = 5 \), \( b_o = 0.96 \) and \( c_o = 1.2 \).

So the nominal values of robot components are:

\[
M_o(q) = \begin{bmatrix}
0.96 & 0.96 + 1.2 \cos(q_1) \\
0.96 + 1.2 \cos(q_1) & 5.96 + 2.4 \cos(q_1)
\end{bmatrix}
\]

\[
C_o(q, \dot{q}) = \begin{bmatrix}
0 & 1.2 \dot{q}_2 \sin(q_1) \\
-1.2(\dot{q}_1 + \dot{q}_2) \sin(q_1) & -1.2 \dot{q}_1 \sin(q_1)
\end{bmatrix}
\]

\[
G_o(q) = 9.81 \begin{bmatrix}
1.2 \cos(q_1 + q_2) \\
5 \cos(q_1) + 1.2 \cos(q_1 + q_2)
\end{bmatrix}
\]

A mechanical tool attached to the robot hand of 0.5Kg, then the robot and tool mass will be \( m_2 = 2\text{Kg} \). The constraint robot is required to move along the constraint surface, tracking the time varying trajectory while simultaneously exerted a desired force on the constraint surface.

The time response of the motion tracking error \( e_1(t) \) with and without ANFIS control is shown in Fig.(2). Fig.(3) shows the force tracking error \( e_2(t) \) of the constrained robot with and without ANFIS.
Fig. (2) Motion tracking error with and without ANFIS

Fig. (3) Force tracking error with and without ANFIS

References


